Making Actions in the Proving Process Explicit, Visible, and "Reflectable"

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Legacy of R.L. Moore 2010

Outline of Talk

- The need for a supplement in beginning real analysis
- Our theoretical framework
- Supplement description
- Surprising student difficulties
- A supplement proof
- Effect on the students

The Need for a Supplement

- The real analysis course is a 3-credit junior level class which serves three populations
  - Math majors.
  - Pre-service secondary math teachers.
  - Graduate students needing remediation.
- In an interview, the real analysis teacher said that the course tries to be all things to all students, which is virtually impossible in three hours a week.

- When asked why she had invited us to provide a supplement the teacher said, “In my opinion students learn to do proofs by doing proofs and not [by] reading them or doing exercises” and that this cannot always be done in the normal class setting.
- She went on to mention our “proofs” course designed to improve graduate students’ proving abilities. She thought the same thing would be helpful for the real analysis students.

Our Theoretical Framework

- We view the proving process as a sequence of mental and physical actions.
- Some actions, such as looking up a definition, drawing a sketch, or focusing on a particular part of the proof, are *not* easily noticed or visible in the final written proof.

- Making such actions, and the reasons for them, explicit and visible facilitates reflection and the autonomous enactment of future similar actions.
- Some repeated actions in the proving process, when paired with triggering situations, can become automated. We call such (small) lasting mental structures, *behavioral schemas*.

(Selden, McKee, & Selden, 2010)

- Enacting behavioral schemas does not require conscious processing and reduces the burden on working memory. This allows working memory to be better applied to the problem-centered parts of proof construction.

(Selden & Selden, 2009)

- Changing a detrimental behavioral schema requires more than just understanding the need for the change.

(Selden, McKee, Selden, 2010)

- This perspective is consistent with that of psychologists like Bargh (1997) who discuss the automated nature of much of everyday life.
- However, to our knowledge, they do not employ a theoretical framework such as we have described.

Supplement Description

- We are describing mainly the second iteration of the supplement, design experiment.

  (Cobb, et al., 2003)

- The students who attended the supplement did so on a voluntary basis and every effort was made to conduct it at a time when almost every student in the real analysis course could attend.
- The supplement met once a week for 75 minutes, a total of one-third of the class time for those students who chose to attend.

- Each week, the real analysis teacher choose a homework problem to be graded very carefully.
- The supplemental teachers worked the problem, noting the actions. Then they selected or wrote a theorem that used many of the same actions but that was *not* a template problem.
- Students who attended the supplement co-constructed its proof with guidance.

- One of the supplement teachers wrote the theorem on the board. Then the students, or teacher if need be, offered suggestions about which actions to do next.
- For each suggested action, such as writing down a definition or drawing a sketch, a student was asked to carry out the action on the board.
- The goal was to have students reflect on what occurred and later to perform these actions, or similar actions, autonomously.

- Every student was encouraged to participate in co-constructing the proof although not every student could carry out every action.
- Class discussion and questions were actively encouraged.
- At the end of each supplemental class, students were given a handout that went through the proof and described the actions – a hypothetical proof co-construction trajectory.

(Simon, 1995)

- The supplement was videotaped and field notes were taken.
- The supplement teachers and the real analysis teacher met following each supplemental class to review what happened and plan for the next supplemental class.
- The real analysis teacher used misconceptions or difficulties that occurred during the supplement to inform her instruction. Further, she pointed out the actions in her lectures to reinforce the supplemental instruction.

Surprising Student Difficulties

- Starting with the hypothesis rather than looking at the conclusion to see what is to be proved.
- Unable to make an appropriate sketch at the appropriate time in the proof.
- Not turning the pages of their books or notes to find the appropriate definitions, theorems, etc.
- Unable to copy a definition accurately.
- Difficulty altering the notation in a definition or theorem to match the current proof.

A Supplement Proof

- Theorem from Supplement: Let $\{a_n\}$ and $\{b_n\}$ be sequences, both converging to P. If $\{c_n\}$ is the sequence given by $c_n=a_n$ when n is even and $c_n=b_n$ when n is odd, then $\{c_n\}$ converges to P.
- Theorem from Class: Show that $\{a_n\}$ converges to A if and only if $\{a_n- A\}$ converges to 0.

- Definition of Convergence - A sequence $\{a_n\}$ converges to a real number A iff for each $\varepsilon>0$ there is a positive integer N such that for all $n>N$ we have $|a_n-A|<\varepsilon$.

- Proof of Supplement Theorem:

  Let $\{a_n\}$ and $\{b_n\}$ be sequences and P be a number so that $\{a_n\}$ and $\{b_n\}$ converge to P. Suppose $\{c_n\}$ is the sequence given by $c_n=a_n$ when n is even and $c_n=b_n$ when n is odd.

  Let $\varepsilon>0$.

  As $\{a_n\}$ converges there exists an N_a such that for all $i>N_a$, $|a_i-P|<\varepsilon$.

  As $\{b_n\}$ converges there exists an N_b such that for all $j>N_b$, $|b_j-P|<\varepsilon$.

  Let $N=\max\{N_a, N_b\}$.

  Let $n>N$.

  Case 1: Suppose n is even. Then $|c_n-P|=|a_n-P|<\varepsilon$.

  Case 2: Suppose n is odd. Then $|c_n-P|=|b_n-P|<\varepsilon$.

  In either case $|c_n-P|<\varepsilon$.

  Therefore $\{c_n\}$ converges to P.

Actions in the Proof

- Write the first line.
- Write the last line.
- Unpack the conclusion.
  - Write the appropriate definition on scratch work.
  - Change the notation to fit the problem.
- Set-up the proof leaving appropriate spaces.
- Find N.
- Recognize the cases.
- Complete the proof including any necessary algebra.

Effect on the Students

- In describing the attempts of supplement students to produce a proof on exams, the real analysis teacher said "I would see the first line, I would see the last line… I can see the technique… some more obvious than others but most definitely it was on the test."
- As the semester progressed, supplement students knew what to do next with less prompting or help.
- Preliminary evaluation of homework papers indicates students who attended the supplement wrote their proofs in a more concise, clear manner.

- The interviewed students all responded very positively concerning the supplement and what they learned therein.
- When asked how the supplement impacted how they construct proofs in their current courses they replied:
  - Knowing where to start;
  - Knowing how to unpack the conclusion;
  - Knowing how to use definitions;
  - Knowing how to use “fixed, but arbitrary.”

References

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Thank you.

Comments/questions?